How to make a wormhole

Understanding global AdS_2

Juan Maldacena

Based on work with Xiaoliang Qi .

Also on previous work:
- Gao, Jafferis, Wall
- JM, Douglas Stanford and Zhenbin Yang.
- Ioanna Kourkoulou and JM

Motivations

- Simple procedure for making the TFD state in SYK.
- Is there an operator to state mapping in 1d CFT's , or nearly-CFTs
- Can we see the consequences of conformal symmetry in the spectrum ?
- Can we understand nearly-AdS_2 with a global time translation isometry ?

Sachdev, Ye, Kitaev model (SYK)

Quantum mechanical model, only time.

$$\{\psi_i, \psi_j\} = \delta_{ij}$$ N Majorana fermions

$$H = \sum_{i_1, \cdots, i_4} J_{i_1 i_2 i_3 i_4} \psi_{i_1} \psi_{i_2} \psi_{i_3} \psi_{i_4}$$ $(H_q = J_{i_1 \cdots i_q} \psi^{i_1} \cdots \psi^{i_q})$

random couplings

$$\langle J^2_{i_1 i_2 i_3 i_4} \rangle = J^2 / N^3$$ J = single dimension one coupling.

N large, strong coupling 1 << (time) J << N (still exponentially many energy levels)

Two copies of SYK + Interaction

$$H = H_L + H_R + \mu \sum_i \psi_L^i \psi_R^i$$

We typically want $\mu/J \ll 1$.

We have a relevant deformation.

We expect to flow to gapped system. Indeed that is what happens.

The point will be to study properties of this gapped deformation.

We will see that its properties are largely determined by the nearly conformal symmetry of the SYK model.

We will also argue that the ground state of the model is close to the TFD state.

$$|TFD\rangle = \sum_n e^{-\beta E_n/2} |\bar{E}_n\rangle_L |E_n\rangle_R$$

Leading order equations

$$\partial_{t_1} G - \Sigma * G = \delta(t_{12})$$

$$\Sigma(t_1, t_2) = J^2 [G(t_1, t_2)]^{q-1}$$

For usual SYK

$$\partial_{t_1} G_{LL} - \Sigma_{LL} * G_{LL} - \Sigma_{LR} * G_{RL} = \delta(t_{12})$$

$$\partial_{t_1} G_{LR} - \Sigma_{LL} * G_{LR} - \Sigma_{LR} * G_{RR} = 0$$

$$\Sigma_{LL}(t_1, t_2) = J^2 [G_{LL}(t_1, t_2)]^{q-1} \ ,$$

$$\Sigma_{LR}(t_1, t_2) = \underline{\mu \delta(t_{12})} + J^2 [G_{LR}(t_1, t_2)]^{q-1}$$

Two coupled SYK systems

Solving the equations

- At low energies
- Numerically
- Analytically in the large q regime

We will now describe these three approaches

Solving the equation using the low energy action

$$\frac{1}{N} \ll \frac{\mu}{J} \ll 1$$

$$\partial_{t_1} G_{LL} - \Sigma_{LL} * G_{LL} - \Sigma_{LR} * G_{RL} = \delta(t_{12})$$
$$\partial_{t_1} G_{LR} - \Sigma_{LL} * G_{LR} - \Sigma_{LR} * G_{RR} = 0$$
$$\Sigma_{LL}(t_1, t_2) = J^2 [G_{LL}(t_1, t_2)]^{q-1} \ ,$$
$$\Sigma_{LR}(t_1, t_2) = \mu \cancel{\delta(t_{12})} + J^2 [G_{LR}(t_1, t_2)]^{q-1}$$

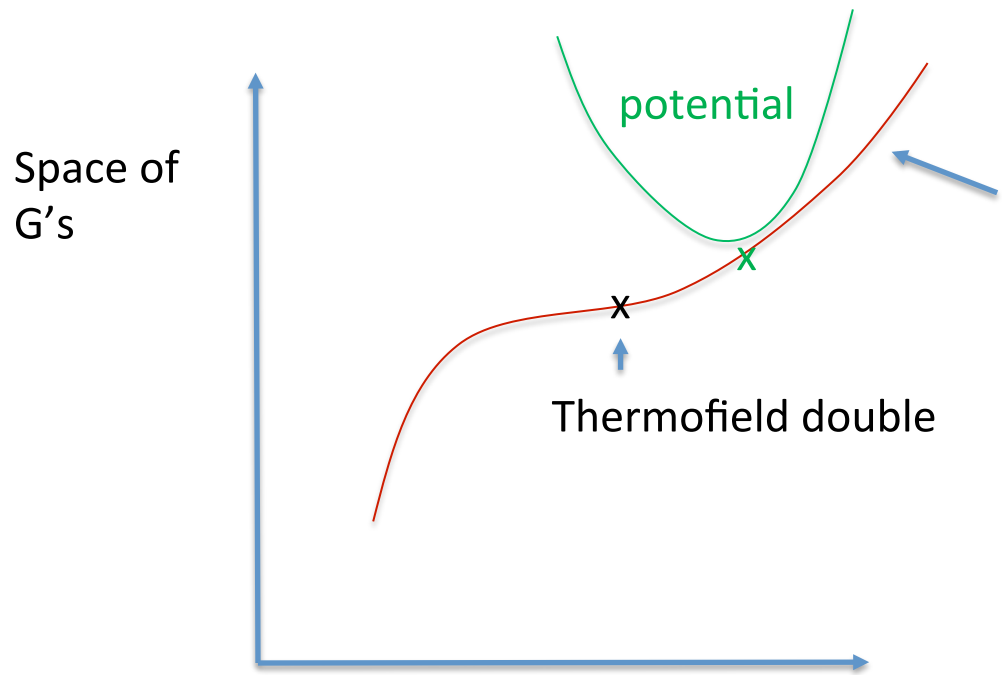
Same as the equations for two decoupled SYK models.

Guess a solution = TFD state

$$H = H_L + H_R + \mu \int du \sum_i \psi_L^i(u) \psi_R^i(u)$$

Make correlations large

The low action valley



$$G = G_f = \left[\frac{f'(t_1) f'(t_2)}{(f(t_1) - f(t_2))^2} \right]^{\Delta}$$

Forget about the degrees of freedom orthogonal to the valley and evaluate the interaction term along the valley

$$\mu \sum_i \psi_L^i \psi_R^i \to N\mu \int G_f(t,t) \longrightarrow N\mu \left[\frac{t_L'(u) t_R'(u)}{\cos^2 \frac{t_L(u) - t_R(u)}{2}} \right]^{\Delta}$$

The effective action along the valley

$$S = N\int du\left\{-\frac{\alpha_S}{\mathcal{J}}\left(\{\tan\frac{t_l(u)}{2},u\}+\{\tan\frac{t_r(u)}{2},u\}\right)+\mu\frac{c_\Delta}{(2\mathcal{J})^{2\Delta}}\left[\frac{t_l'(u)t_r'(u)}{\cos^2\frac{t_l(u)-t_r(u)}{2}}\right]^\Delta\right\}$$

Why the tangent ?
We could have chosen the original f , but the solution would be more complicated.
We will understand later why this was a good guess.

$$t_l(u) = t_r(u) = t'\,u\ , \qquad t' = \text{constant}$$

This is a solution for any t' .

We also need to require invariance under the SL(2) transformations transforming t.
These are gauge symmetries.

Imposing invariance under t translations

$$\frac{\alpha_s t'}{\mathcal{J}} = \frac{\mu}{\mathcal{J}} \left(\frac{t'}{\mathcal{J}} \right)^{2\Delta - 1} \longrightarrow \left(\frac{t'}{\mathcal{J}} \right)^{2-2\Delta} \sim \frac{\mu}{\mathcal{J}}$$

If μ is small → t' is small, justifying the approximation. Note that Δ < ½ in SYK.

$$\text{for} \quad 0 < \Delta < \frac{1}{2} \,, \qquad \mu \ll t' \ll J$$

We can compute the energy.

$$E \propto -N \frac{\alpha_s t'^2}{\mathcal{J}} \left(\frac{1}{\Delta} - 1 \right)$$

It is lower than the ``ground state" energy of the decoupled SYK models.

We can obtain the same equation by the following variational method. We consider a TFD at temperature β and evaluate the vacuum expectation value of the full Hamiltonian and then minimize with respect to β.

$$t' = \frac{2\pi}{\beta}$$

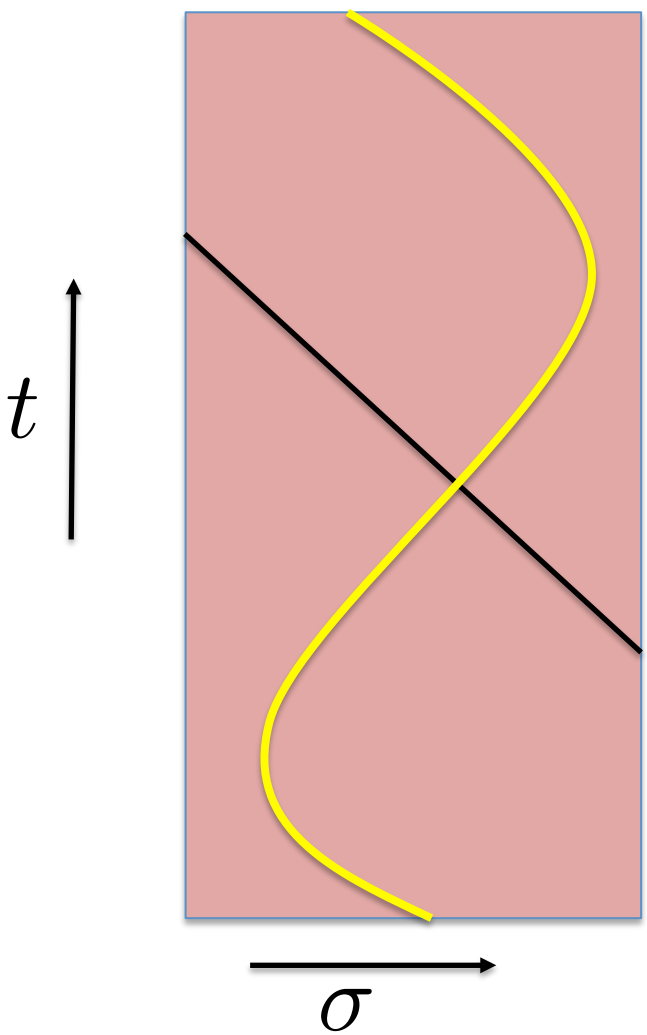
This quantity gives us the relation between the boundary time u and the infrared time t.

After discussing the AdS_2 picture we will remark more on the physical meaning of t.

For now let us just say that the energies are dimensionless, order one numbers when we measure them with respect to t. Translating them to energies measured with respect to boundary time, u, they get their overall scale from t'.

AdS_2 Interlude

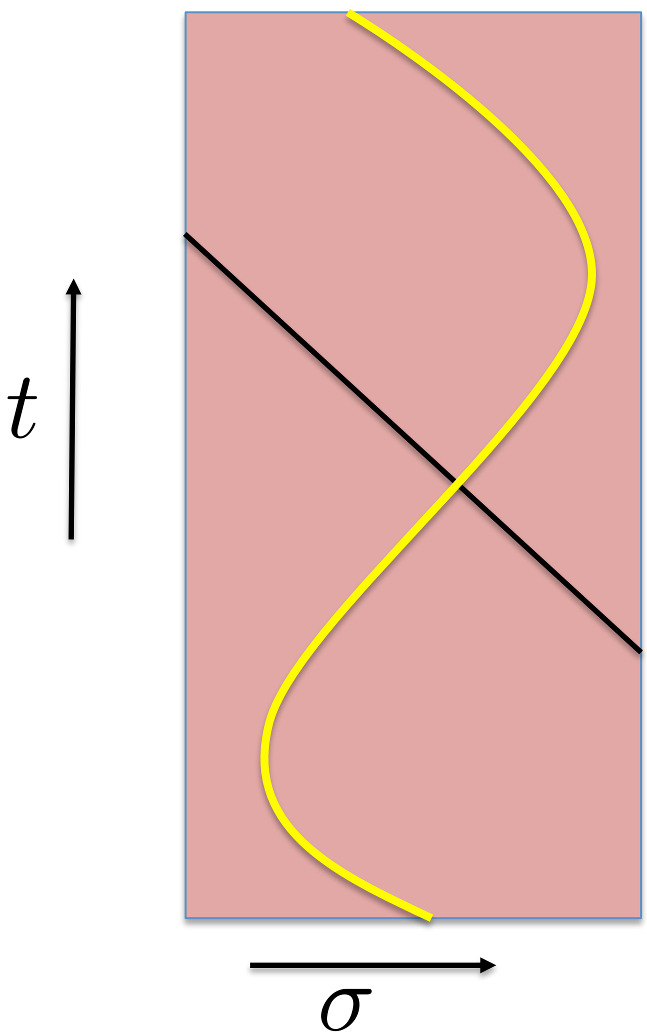
AdS$_2$ - Global coordinates



$$ds^2 = \frac{-dt^2 + d\sigma^2}{(\sin\sigma)^2}$$

- SL(2,R) isometries
- Two boundaries
- Causally connected
- Particle dynamics → oscillatory behavior → gapped spectrum
- Global coordinates

AdS_2 : A traversable wormhole



$$ds^2 = \frac{-dt^2 + d\sigma^2}{(\sin\sigma)^2}$$

- SL(2,R) isometries
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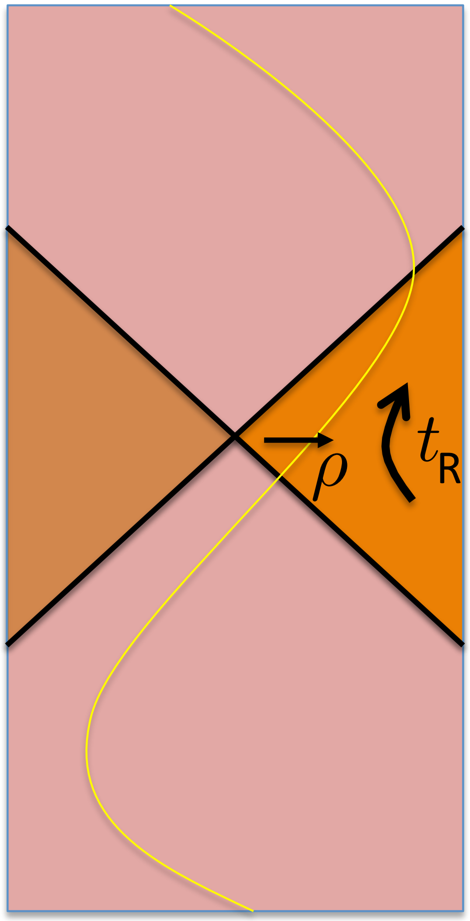
AdS_2 - thermal (Rindler) coordinates



$$ds^2 = -dt_R^2 \sinh^2 \rho + d\rho^2$$

- Two boundaries
- Cover only a portion of AdS_2
- <u>Causally disconnected</u>

t, t_R are conjugate to two different elements of the SL(2,R) isometries of AdS_2

Non-traversable wormhole

AdS_2 vs $NAdS_2$ asymptotic boundary conditions

- Exact AdS_2 boundary conditions do not make sense

JM, Michelson, Strominger

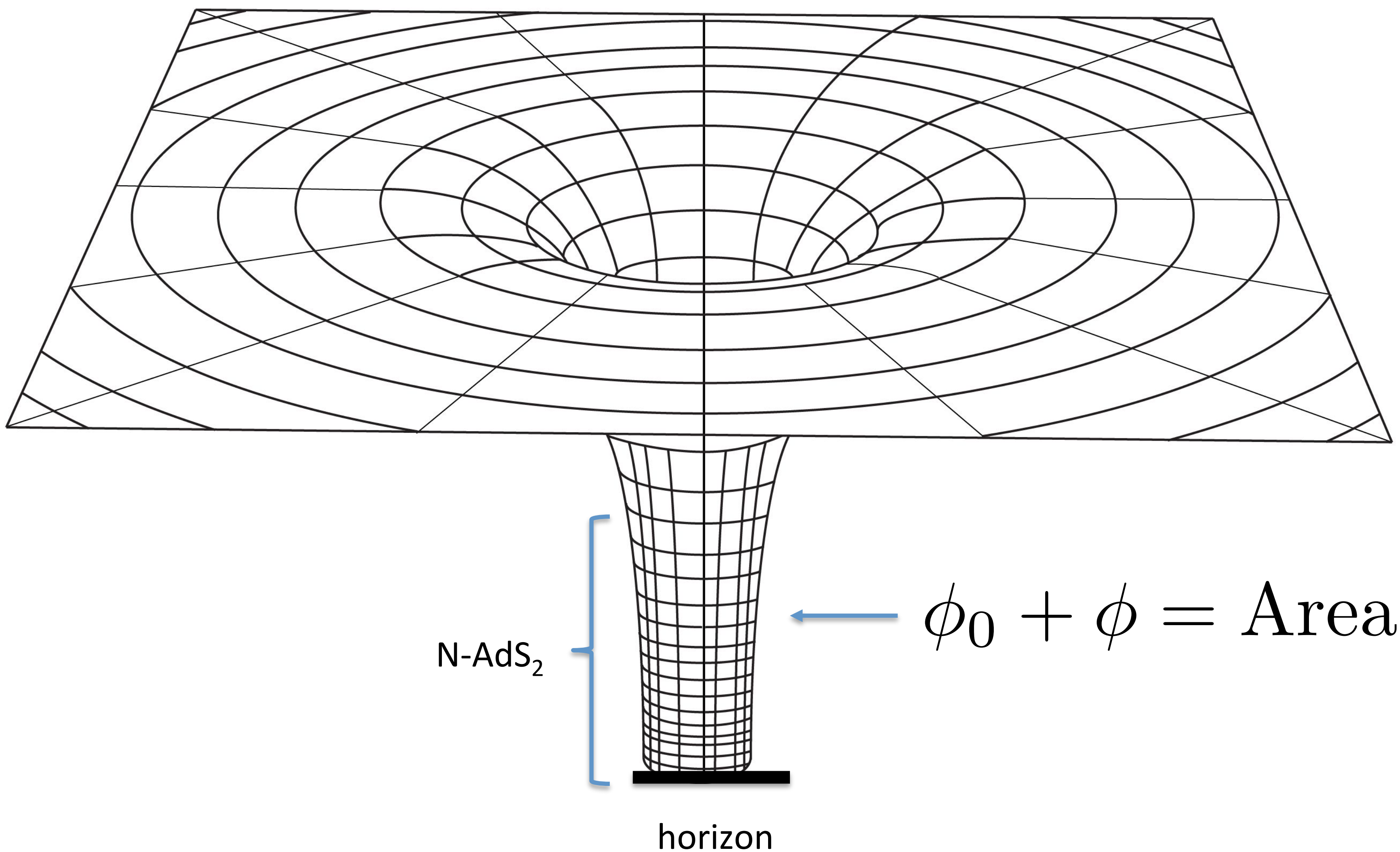
- Need to break some of the AdS_2 isometries slightly

Almheiri Polchinski

- We should think about nearly-AdS_2

- Nearly AdS_2 with t_R-isometry → TFD of Nearly CFT_1
- Nearly AdS_2 with t-isometry → What we are discussing here.

First recall some facts about nearly AdS_2 boundary conditions…



$\phi_0 + \phi = \text{Area}$
N-AdS$_2$
horizon

Nearly AdS_2 gravity

Keep the leading effects that perturb away from AdS_2

Teitelboim Jackiw
Almheiri Polchinski

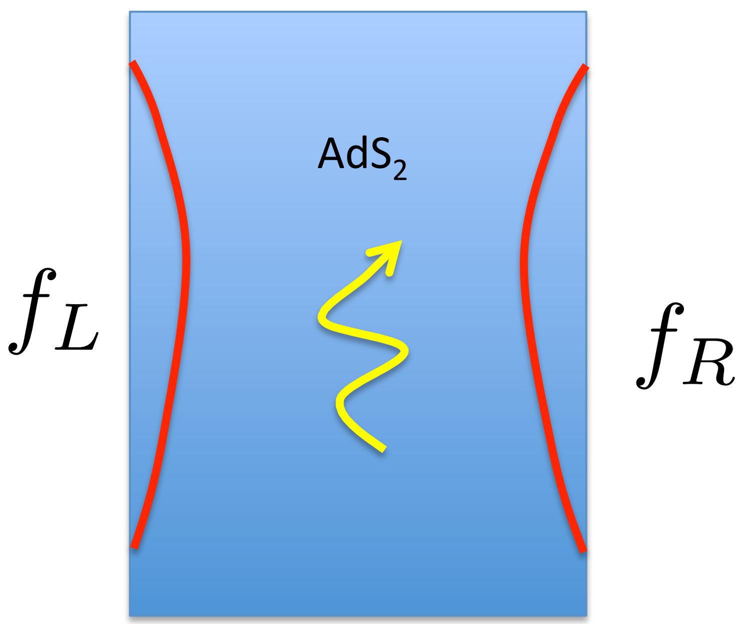
$$\int d^2x \sqrt{g}\,\phi(R+2) + \phi_0 \int d^2x \sqrt{g} R$$

Comes from the area of the additional dimensions.

Ground state entropy

No bulk excitations → only "boundary gravitons" → location of the physical boundary in AdS_2
Exactly the same action as the Schwarzian.

Gravitational dynamics



$$\int \phi(R+2)$$

Rigid AdS$_2$

Physical boundary given by dilaton

Dynamics is in the position of the boundary. = Schwazian action

Boundary graviton: encodes the motion of the boundary.

$$(H_{f_L} \times H_{\text{bulk}} \times H_{f_R})/SL(2,R)$$

In the SYK case we have something similar

$$(H_{f_L} \times H_{\text{bulk}} \times H_{f_R})/SL(2,R)$$

Excitations in the directions normal to the "valley"
Described by conformal invariant methods.

The interaction between the two boundaries

- Nearly-AdS_2 gravity
- Plus matter
- Plus boundary conditions connecting the two sides (as in Gao-Jafferis-Wall)

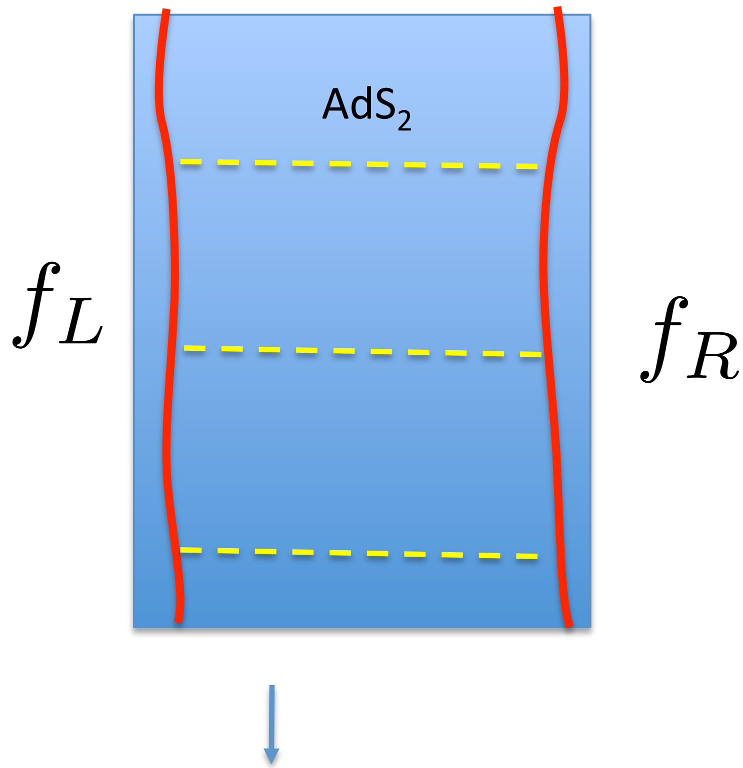
$$S_{int} = \mu \int du \chi_L(u) \chi_R(u)$$

u is proper length along the boundary, or boundary time.

- This generates negative null energy and allows for a solution with the global time isometry, where ϕ grows towards both boundaries

AdS$_2$ gravity +

Interaction

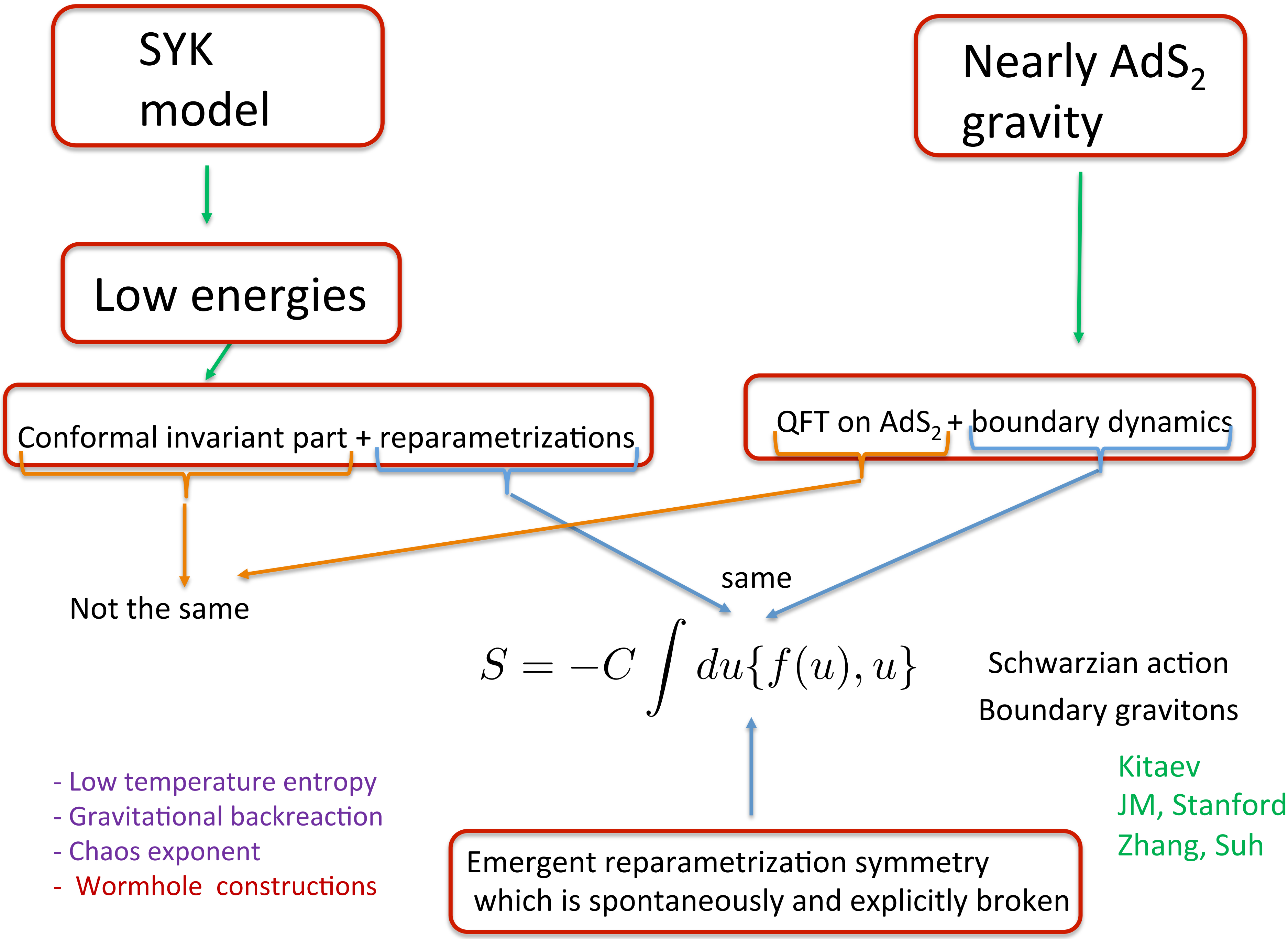


$$S = \frac{N\alpha_S}{J}\int du \{f_L(u),u\} + \{f_R(u),u\} + N\mu \int du \left[\frac{f_L'(u) f_R'(u)}{|f_L(u) - f_R(u)|^2}\right]^\Delta$$

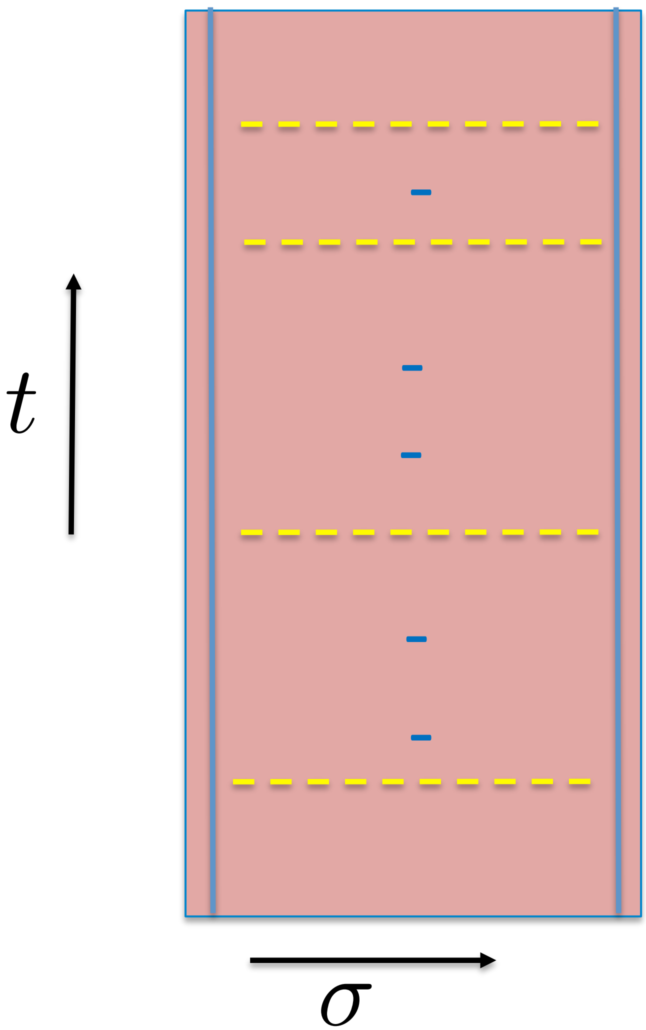
+ Global SL(2,R) gauge symmetry → set total SL(2,R) charge to zero.

$$f(u) = \tan\left(\frac{t(u)}{2}\right)$$

- The SYK model has some properties in common with nearly AdS_2 gravity.
- It has the same gravitational dynamics.
- This dynamics is expected to be universal for any system with an almost conformal symmetry in the IR (which is not integrable).

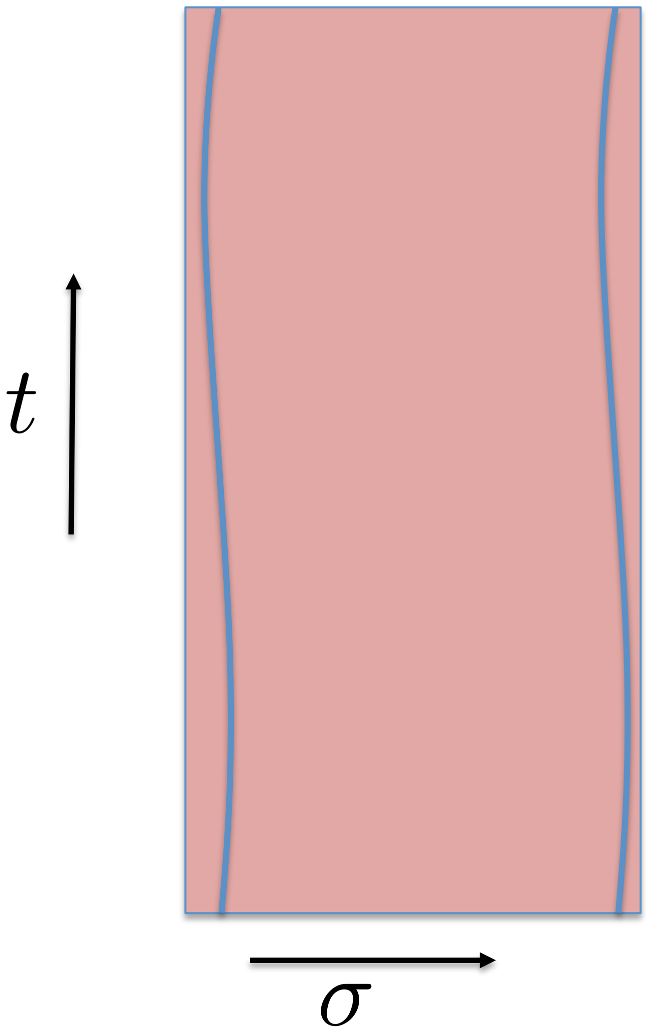

SYK model
Nearly AdS_2 gravity
Low energies
Conformal invariant part + reparametrizations
QFT on AdS_2 + boundary dynamics
Not the same
same
$S = -C \int du \{ f(u), u \}$
Schwarzian action
Boundary gravitons
Kitaev
JM, Stanford
Zhang, Suh
- Low temperature entropy
- Gravitational backreaction
- Chaos exponent
- Wormhole constructions
Emergent reparametrization symmetry
which is spontaneously and explicitly broken

Nearly AdS_2



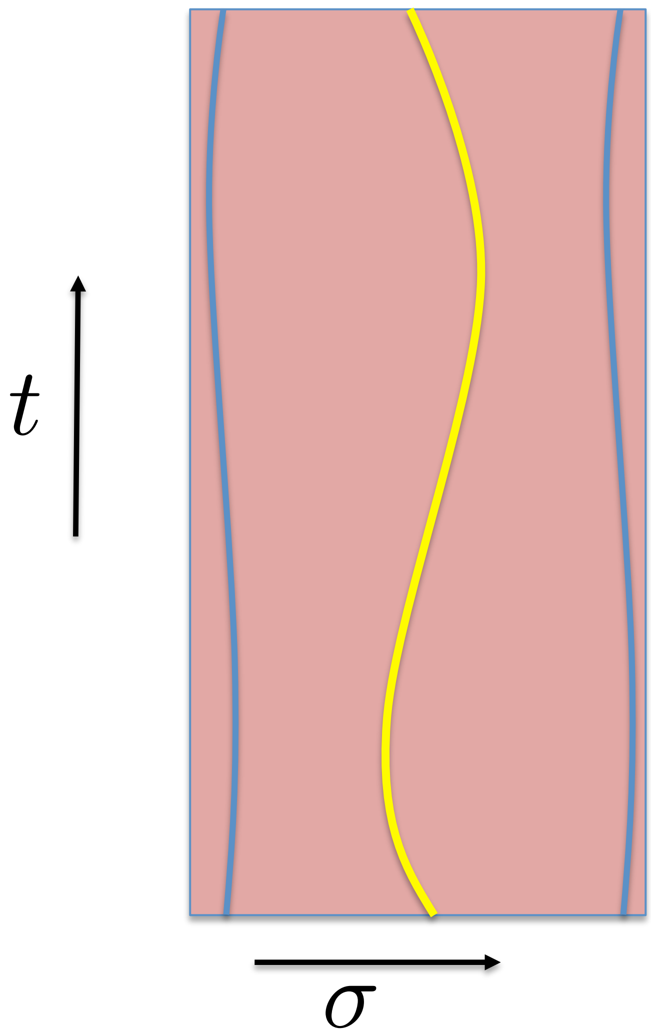
Casimir force due to the boundary conditions connecting the left and right sides → attractive force between the two boundaries.

Nearly AdS_2



Small oscillations around equilibrium position.

Nearly AdS$_2$



Adding matter.
Matter leads to a conformal spectrum to leading order.

Universal Spectrum

Recall the solution $\left(\frac{t'}{\mathcal{J}}\right)^{2-2\Delta} \sim \frac{\mu}{\mathcal{J}}$ Same here from AdS

Field in AdS_2 corresponding to a boundary operator of dimension Δ →

$$E = E_u = t'(\Delta + n)$$

Spectrum governed by conformal symmetry
(like in higher dimensional global AdS)

Schwarzian, or dynamical boundary degree of freedom, (boundary graviton) →
For small perturbations: one harmonic oscillator with energy

$$E = t'\sqrt{2(1-\Delta)}(n + \frac{1}{2})$$

Stable equilibrium

Same energy scale as the particles inside

This part is not conformal invariant.

Spectrum is completely fixed !

In SYK we can also solve the theory beyond the low energy limit.

Methods to analyze the equations

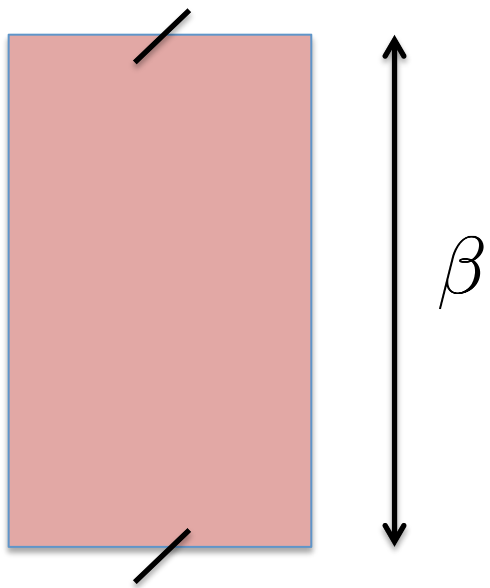
- Numerical
- Large q approximation → analytic solution.

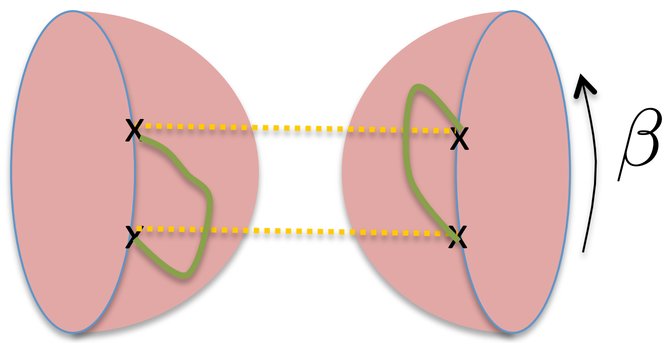
- We can now solve the equations not limited to the small $\frac{\mu}{J}$ approximation.

- It is also interesting to study the finite temperature situation.

Finite temperature and Hawking Page phase transition.

- Two possible finite temperature configurations in Euclidean space



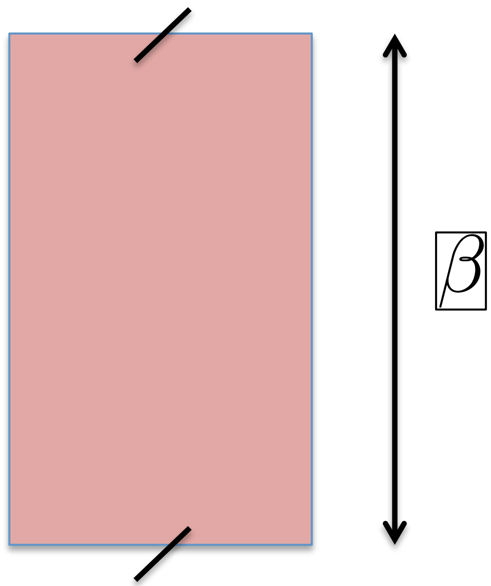


$$\log Z/N \sim 2S_0 + \frac{1}{\beta J} + \mu^2(\beta J)^{2\Delta}$$

"ground state" entropy contribution

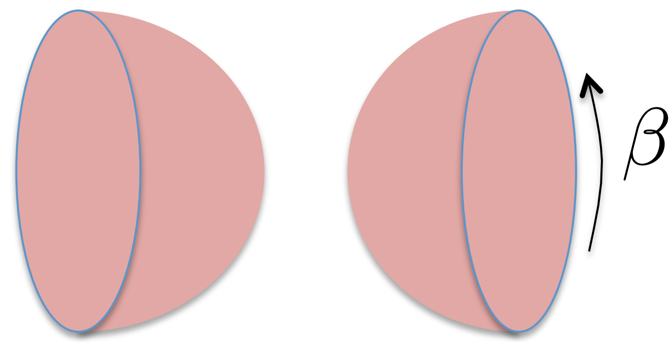
$$\log Z/N \sim -\beta(T')^2 + e^{-\beta T'\Delta} \sim -\beta\left(\frac{\mu}{J}\right)^{\frac{1}{(1-\Delta)}} + e^{-\beta T'\Delta}$$

Finite temperature and Hawking Page phase transition.



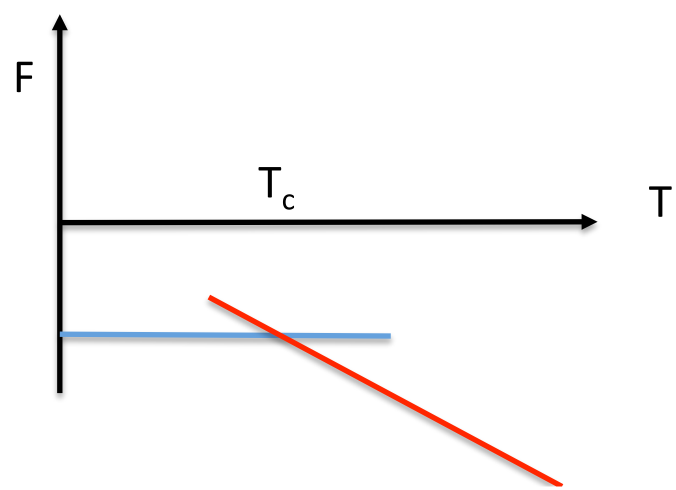
$$\log Z/N \sim \beta \left(\frac{\mu}{J}\right)^{\frac{1}{(1-\Delta)}}$$

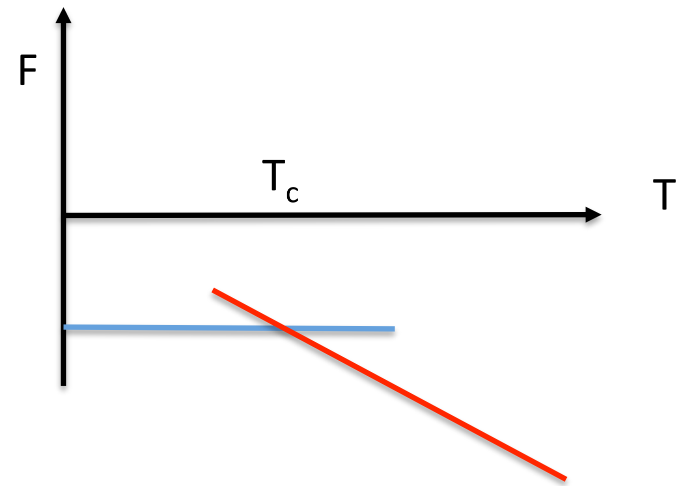
Low temperature



$$\log Z/N \sim 2S_0$$

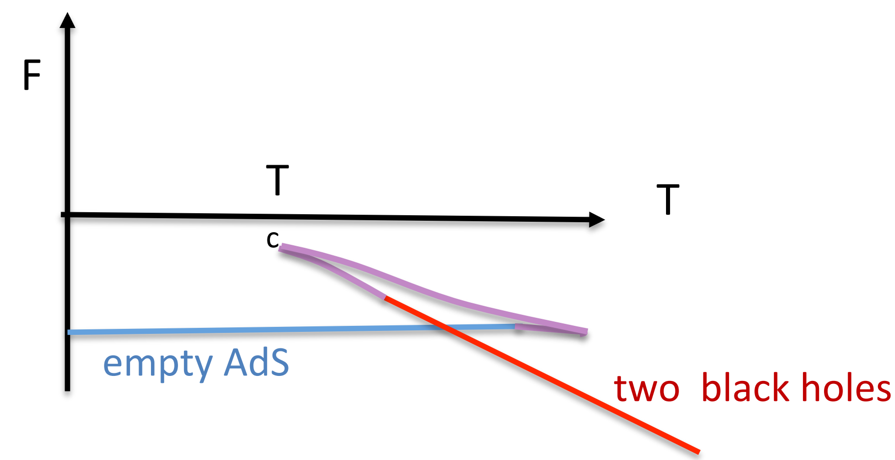
High temperature





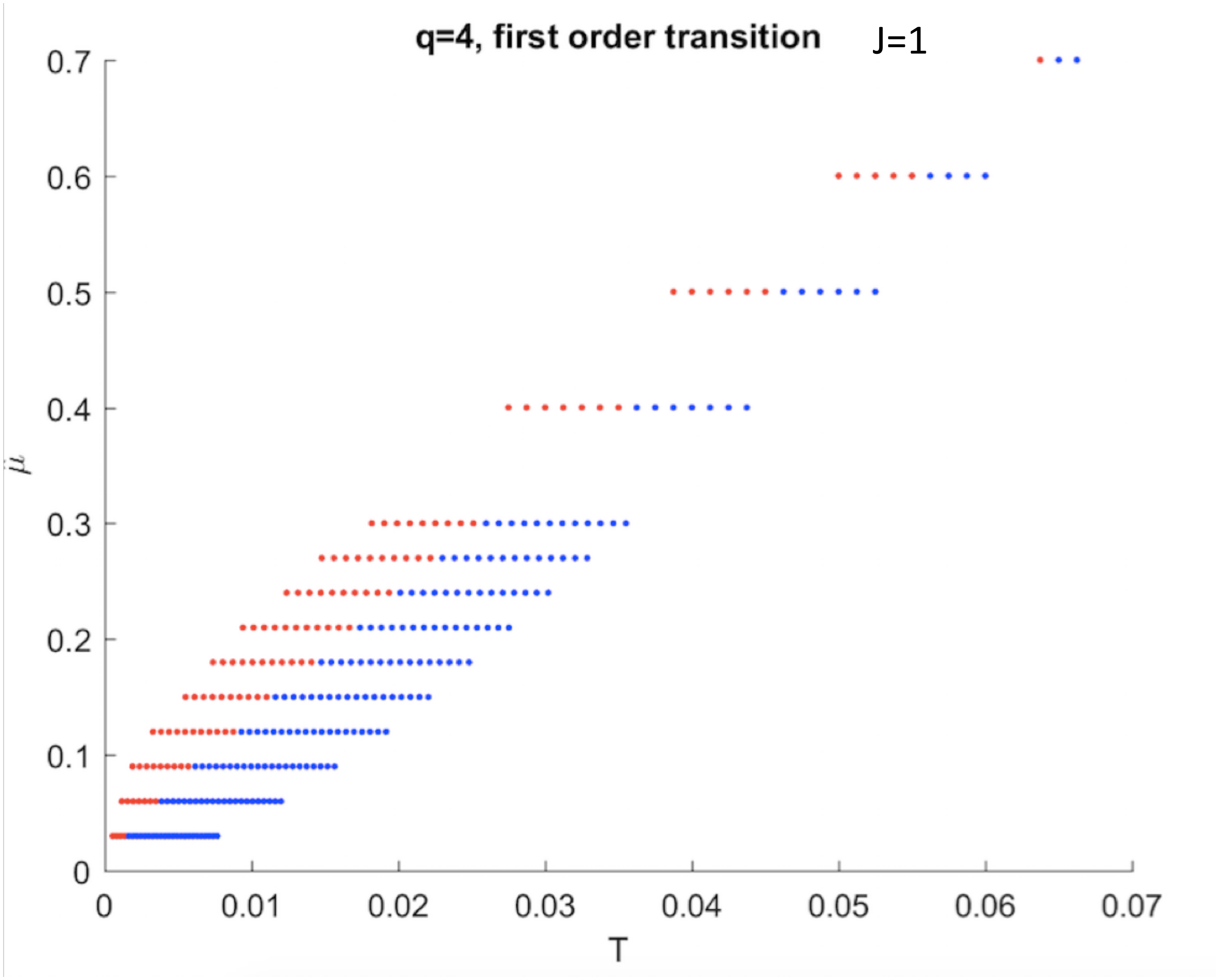
- Both phases can be described using the conformal approximation + Schwarzian correction, or as nearly AdS_2 gravity configurations plus further small corrections.
- Phase transition happens when both approximations are valid.
- But the underlying conformal solutions really <u>different</u> configurations. The SL(2,R) gauge symmetries act differently.

- At large q, analytically one can find



- In the cannonical ensemble: 1st order phase transition (like Hawking-Page).
- In the microcannonical ensemble → continuous behavior.
- Continuous connection between the phase with no black holes and the one with a ``small black hole''

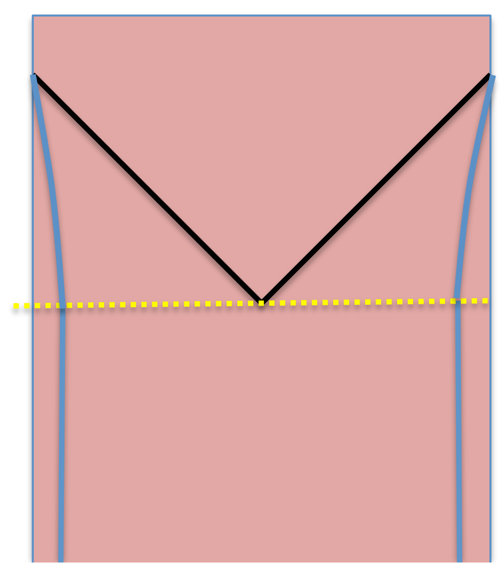
Numerical Analysis



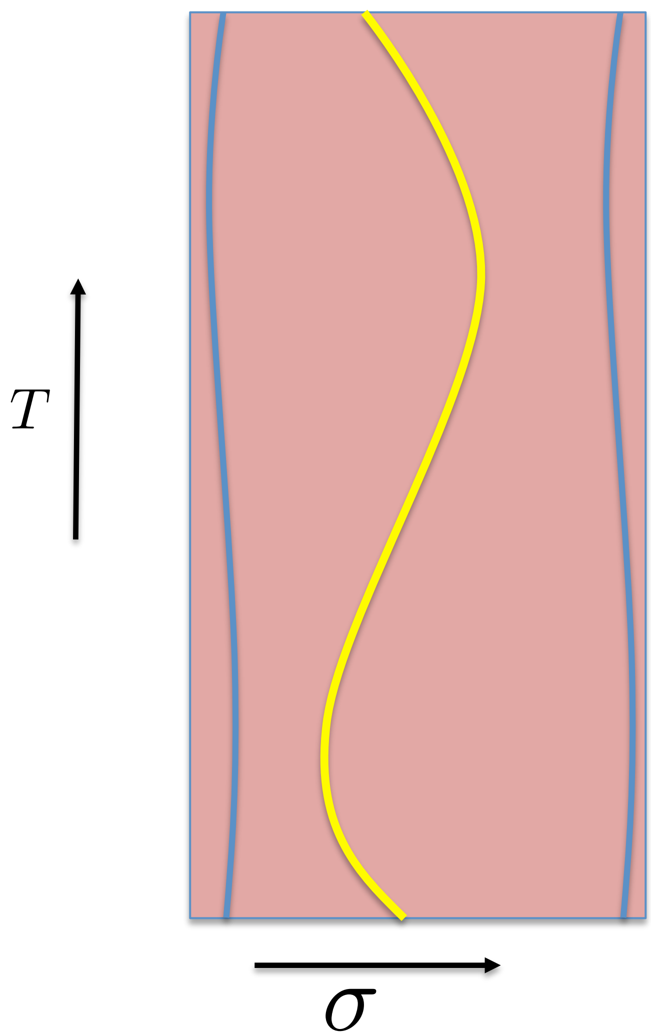
Making the TFD

- Create two SYK systems.
- Couple term. $\mu \neq 0$
- Couple them further to a heat sink and let them cool down to find its ground state.
- At t=0, turn off the left-right coupling. $\mu = 0$

- → Get a state that is close to the TFD.



Matter oscillations

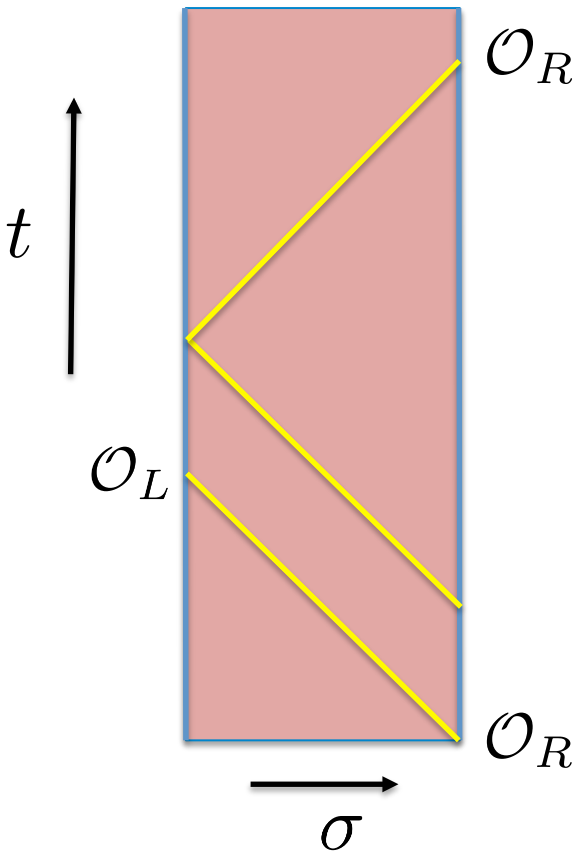


Bulk matter particle → Oscillations → excitation goes from mostly from the left SYK to mostly on the right SYK.

Governed by conformal symmetry.

Correlation Functions

$$\langle \mathcal{O}_L(\tilde{u}_1)\mathcal{O}_L(\tilde{u}_2)\rangle = e^{-i\pi\Delta}\left[\frac{t't'}{\sin^2\frac{t'(u_1-u_2-i\epsilon)}{2}}\right]^\Delta , \qquad \langle \mathcal{O}_L(\tilde{u}_1)\mathcal{O}_R(\tilde{u}_2)\rangle = \left[\frac{t't'}{\cos^2\frac{t'(\tilde{u}_1-\tilde{u}_2)}{2}}\right]^\Delta$$



They are singular when the times can be joined by a light ray in the bulk

The singularity is removed by two effects.

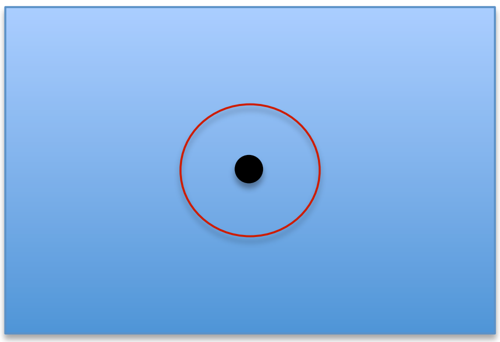
a) the UV deformation away from the conformal limit.

b) Backreaction → Inclusion of the boundary dynamics.

Both for AdS_2 and SYK models. Oscillations back and forth between the left and right systems.

State/operator mapping

2d CFT



“radial” quantization → operators on plane
to sates on the circle x time.

1d nearly CFT

Naïve:
Operators → states in two decoupled copies

By introducing the small coupling, we can
study the spectroscopy of these states
in detail.

Additional comment

- Even if the left and right couplings are different, we still get a solution which looks connected.
- The energy gap becomes smaller if the couplings are different. It decreases as the couplings get less correlated.
- We do not need perfect matchings of energies to build a state that is close to the TFD double, or that behaves as if the gravity dual was connected.

$$|TFD\rangle = \sum_n e^{-\beta E_n/2} |\bar{E}_n\rangle_L |E_n\rangle_R$$

Conclusions

- We can analyze two coupled SYK models
- They have a ground state close to the TFD and properties determined by the nearly conformal symmetry.
- A similar problem in nearly-AdS_2 gravity leads to traversable wormholes.
- Discussed thermal aspects and the phase transition.
- Realized a state close to the TFD as the ground state of the coupled system.